Seismic response of symmetric structures having unbalanced yield strengths in plan

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ABSTRACT

Symmetric structures, with coincident centers of mass and stiffnesses in plan, can be excited torsionally in the inelastic domain due to unbalanced distribution of their yield strength. Under some circumstances, this can produce a magnification of the otherwise expected ductility demand of the lateral load resisting structural element having the lesser strength. A parametric study has been conducted to investigate the circumstances where this amplification becomes significant, and results of this research are presented herein.

INTRODUCTION

Current seismic retrofitting strategies emphasize the need to eliminate the eccentricities in plan of existing structures by adding new lateral load resisting structural elements (LLRSEs), and by calibrating the stiffnesses of the new elements such as to minimize these eccentricities. While the resulting retrofitted structures become symmetric in the elastic sense, the new structural elements used are often of a different type than the existing ones. This results in dissimilar yield strengths between the various LLRSEs, and a transient state of torsional response is consequently expected to develop when the structure will be excited in the inelastic range. It is noteworthy that such dissimilarities can also be present in many types of new or existing structures, simply as a consequence of other engineering or architectural decisions; thus the particular structural behavior described above can be equally attributable to various other causes.

While other researchers have investigated the effect of torsional instability of symmetric systems *[Tso and Asmis 1971, Tso 1975, Antonelli et.al. 1981, Pekau and Syamal 1984]* and the effect of seismic wave motions characteristics in exciting torsional modes in otherwise symmetric structures *[Newmark 1969, Awad and Humar 1984]*, little research has focused

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on elastically symmetric structures having considerable dissimilarities in yield strength and force-displacement relationships. The results reported herein are from research conducted at the University of California, Berkeley, from 1985 to 1987 *[Bruneau and Mahin 1987]*. In the time elapsed since, results from similar research has been published by Pekau and Guimond *[1988]*. Both pieces of work are complementary as they address the same problem in a different perspective. As the problem of inelastic seismic response of torsionally coupled structures is currently receiving a renewed and considerable attention by the research community, it is felt that the present reporting of the original work by the authors is worthwhile, and long overdue.

In a first approach to this broad subject, the present study was limited to the consideration of idealized LLRSEs of identical force-displacement relationship having dissimilar yield strengths. The results of a parametric study of different simple monosymmetric initially symmetric structures having two LLRSEs are presented following.

EQUATIONS OF MOTION AROUND THE CENTER OF MASS

The general equations of motion around the center of mass for single-story torsionally coupled structures are well known and have been derived by others, *[Awad and Humar 1984, among many]*. From these equations are obtained the following parameters generally describing torsionally coupled structures, which are also adopted for this study.

$$\Omega = \omega_\theta / \omega_x = T_x / T_\theta \qquad [1]$$

$$\omega_x^2 = K_x / m \qquad [2]$$

$$\omega_\theta^2 = K_\theta / mr^2 \qquad [3]$$

K_X and K_θ are the structure's translational (along **X**) and rotational (around θ) stiffnesses. The mass of the floor is m, and its radius of gyration r. ω_x and ω_θ are the translational and torsional uncoupled frequencies, T_x and T_θ the corresponding uncoupled periods, and Ω the ratio of those uncoupled frequencies. The reader not familiar with those equations should refer to *Bruneau and Mahin 1987*. For linear elastic perfectly symmetric structures, an uncoupling of the torsional and translational response is possible. This uncoupling would persist until dissymmetric yielding of the LLRSEs occurs, at which point a transient state of torsional coupling is established. The corresponding instantaneous equations of motion could be established *[Bruneau and Mahin 1987]*.

METHODOLOGY OF THE PARAMETRIC STUDY

This parametric study was performed in order to assess the significance of the torsional coupling developing in the inelastic range when symmetric structures consist of LLRSE's of identical force displacement relationship, but dissimilar yield strengths; in particular, the effect of various parameters on the element ductility demand of the initially symmetric structure were investigated when equivalent SDOF systems achieved preselected target ductility values, μ_T.

In this study, simple structures having two LLRSEs equidistant from the center of mass are used. The torsional stiffness of individual LLRSEs is neglected. All floor diaphragms are assumed to be infinitely rigid in their own plane. Elements in the orthogonal direction are ignored for the sake of simplicity. The structure studied is illustrated in Fig. 1. Simple bi-linear inelastic element model was chosen. The introduction of more sophisticated modelling was not warranted at this stage, but due consideration has been given to other non-linear element models elsewhere *[Bruneau and Mahin 1987]*. Providing both LLRSEs are of the same force-displacement relationship type, the element model has been found to have little influence on the conclusions of this study. Strain-hardening was set to 0.5% of the initial elastic stiffness of the elements, making the element model almost elasto-perfectly plastic. The damping was chosen to be of the Rayleigh type, arbitrarily set at 2% of the critical damping for each of the true elastic frequencies of the structures analyzed. For the initially symmetric structures used in this study, the LLRSE's yield level combinations are expressed as "Ry and x(Ry)" (with corresponding yield displacements Δ_Y and $x(\Delta_Y)$, x being a fraction or multiplier of the reference yield strength Ry. For $x \neq 1.0$, the resulting mismatch between the yield strengths of the LLRSEs produces the inelastic torsional response of interest in this study. The strong and weak elements are obviously defined as those having the largest and smaller yield strengths respectively. For $x = 1.0$, the resulting structures constitute equivalent SDOF systems whose inelastic response provides a basis for comparing element ductility demands.

The study was performed for ten values of uncoupled period T_x (0.1, 0.2, 0.3, 0.4, 0.6, 0.8, 1.0, 1.2, 1.6, and 2.0 seconds), six values of the ratio of uncoupled frequencies Ω (0.4, 0.8, 1.0, 1.2, 1.6, and 2.0), two SDOF target ductility levels μ_T (4 and 8), and four element yield combinations ("0.8 and 1.0 Ry", "1.0 and 1.2 Ry", "1.0 and 1.5 Ry", "1.0 and 2.0 Ry"). The equivalent SDOFs used for comparison were all selected to yield at Ry.

The four chosen yield level differences bracket many possible situations. Ultimately, further increasing the difference between the strong and weak elements could lead to permanent elastic response of the strong element with no further changes in element response. Note that although the difference in yield levels are herein assumed to result from the difficulty, or impossibility, in achieving similar stiffnesses and yield levels in different LLRSEs, this difference also implicitly considers the difficulty in accurately predicting the yield strength of some types of structural systems. Further, the intent is to assess the significance of over or under-estimating the yield strength of one element, and consequently, systems of different ultimate translational strengths will be compared in this process.

The following methodology was adopted for the parametric study.

1. Equivalent SDOF systems were selected to have a period equal to the uncoupled translational period T_X, the only period of initially symmetric structures excited before

initiation of yielding. These SDOF were designed such that they shared the same hysteretic characteristics and same yield displacement Δ_Y.

2. Normalized strength factors necessary for each SDOF system to attain target ductilities μ_T of 4 and 8 were calculated for each earthquake record considered. Constant ductility inelastic response spectra were constructed, from which the required strength factors, as a function of SDOF period, were read. Normalized strength factors are defined as:

$$\eta \;=\; R_Y / m\, a_{MAX} \;=\; K_X\, \Delta_Y / m\, a_{MAX} \;=\; \omega_X^{\,2} \Delta_Y / a_{MAX} \qquad [4]$$

where a_{MAX} is the peak ground acceleration of a particular earthquake record, R_Y is the yield strength of the SDOF, and m is the mass of the equivalent SDOF system. For this study, ductility demand μ is defined as the maximum displacement, in absolute value, divided by the yield displacement. For simplicity in this study, peak ground accelerations were scaled as necessary, for fixed values of element model properties, to satisfy the imposed target ductility condition. These steps were taken to ensure that the SDOF structures were insensitive to variations in ground motion intensity. While this departs from a design approach, it ensures that any period-dependency observed in the calculated ductility amplification ratios (see item 4 following) is only attributable to the inelastic torsional coupling phenomena, and not to the seismic input spectral characteristics.

3. For the earthquake excitation levels calculated in the previous step, the same structures were reanalysed considering the unequal element yield strengths. The maximum inelastic element displacements were then calculated, as well as the corresponding element ductility demands. It is noteworthy that, as yield displacement is proportional to yield strength, equal maximum displacements will result in larger ductility for the weak element. Ductility demand of LLRSEs were selected as the response value of interest in this study.

4. The ductility demands calculated for each individual initially symmetric case analyzed above were then divided by the ductility demands obtained from their respective equivalent inelastic SDOF system, to obtain a ratio of the ductilities [indicated "Ductility Amplification Ratios" on all figures herein]. This amplification ratio provides a normalization over the selected target ductilities. It is believed that the ductility amplification ratios for each element of the two-element structures provide the best quantitative measure of the damage sensitivity of the structures.

To provide results mostly independent of the particular characteristics of a single earthquake, five different earthquake records (El Centro 1940 N-S, Olympia 1949 N-S, Parkfield 1966 S16E, Paicoma Dam 1971 N65E, and Taft 1952 N21E) were considered, and the mean, and mean-plus-one-standard-deviation, of response values were calculated.

RESULTS OF THE PARAMETRIC STUDY AND OBSERVATIONS

Figs. 2 and 3 present the results from step 4 above. These plots show the mean ductility amplification ratios of the weak [Fig. 2] and strong [Fig. 3] elements for a target ductility of 4, for the five earthquake records described. Results pertaining to the mean-plus-one-standard-deviation of the ductility amplification ratios, as well as results for a selected target ductility of 8, are presented elsewhere *[Bruneau and Mahin 1987]*.

Considering the nature of ductility measurements in earthquake engineering, and the accuracy expected in ductility prediction of this kind, it might be said that element ductility amplification ratios of 1.25 or less are not considered significant, ductility amplification ratios from 1.25 to 1.5 are considered of moderate importance, and ratios above 1.5 are judged to be of major importance. Following this arbitrary convention, the following can be observed.

1. The weak element ductility amplification ratios for the case "0.8 & 1.0 Ry" are always at least of moderate importance, and often of major importance. This amplification is most severe for cases with small periods or large Ω values (and most significantly a combination of both), with ductility amplification ratios ranging from 2 to 4 for the mean response [Fig. 2]. Amplifications were somewhat expected since the ultimate translational strength of the "0.8 Ry and Ry" structures are less than that of their reference systems; nevertheless, the rather large magnification of weak element ductilities obtained remain impressive.

2. When the yield level of one element is superior to that of the reference SDOF, the weak element ductility amplification ratios are mostly non-affected until Ω becomes larger than 1.6 for the mean response. In that case, the response is also seen to slightly increase along with the yield stresses differentials. The increase in weak element ductility amplification ratios, despite the increased ultimate translational strength of the structures, is surprising. It implies that the added torsional behavior induced by the increase in yield level differential more than overcomes the benefit one might associate with the increase in strength (or balances it in the best case). Increases of 100% are seen for large Ω and large yield level differences, and much larger ductility amplification ratios, often up to 2.5, were observed for single earthquake excitation results. Thus, there is no guarantee that an unbalanced increased strength in a symmetric structure decreases ductility. It should be noted that at some point, further increase in yield level differential would produce no additional change in response for either elements, as the strong element would reach permanently elastic behavior.

3. The strong element ductility amplification ratios are all less than 1.0, except in the "0.8 & 1.0 Ry" case where the lower structures ultimatë translational strengths make larger inelastic deformations also possible in the stronger element. Ductility amplification ratios of moderate importance can be noticed in the case of low periods ($T \leq 0.2$) and low Ω values ($\Omega \leq 0.8$) [Fig. 3]. It is noteworthy that the decrease in strong element ductility amplification ratios occurring with the increase in the ultimate translational strength of

the structures is partly a consequence of the increase of the yield level of the strong element; i.e. an increase in yield level (corresponding to an equivalent increase in the yield displacement), will produce an effective reduction in the ductility demand for a given magnitude of displacement. A value of the strong element ductility amplification ratio below 1.0 reflects that situation; it does not imply that the strong element remains elastic, but simply that ductility demand is less than that of its corresponding SDOF system.

4. The observed ductility amplification ratios are generally independent of target ductility levels. Demonstration of this is presented elsewhere *[Bruneau and Mahin 1987]*.

5. The structural period is seen to have no significant influence on the ductility amplification ratios of initially symmetric structures, except for the "0.8 and 1.0 Ry" case, where weak element ductility amplification ratios are generally larger for structures with small periods [Fig. 2]. This is expected as equivalent SDOF systems have been calibrated to target ductilities with yielding set at Ry. The case "0.8 and 1.0 Ry" having smaller ultimate translational strength than its equivalent SDOF system, the natural tendency of short period structures to have larger response than more flexible structures, typical of earthquakes for the West Coast of the United States, resurfaces.

6. For the methodology followed herein, the element yielding at Ry (i.e. the strong one in the case "0.8 Ry and Ry," and the weak one in the other cases) will always have the same inelastic response as the SDOF system yielding at Ry when $\Omega = 1.0$, and therefore the element ductility amplification ratios will always be 1.0 in that particular case. This rather interesting phenomenon can be accurately predicted by theory, and is explained in great detail elsewhere *[Bruneau and Mahin 1987]*.

7. As seen from Figs. 2 and 3, weak element ductility amplification ratios tend to increase with larger Ω, while strong element ductility amplification ratios tend to decrease accordingly. This can be explained by the lower resistance to angular motion provided by structures with larger Ω values, as explained in more details elsewhere *[Bruneau and Mahin, 1987]*. Obviously, this increase in weak element ductility amplification ratios with Ω would not be observed as consistently when looking at the response under a given earthquake excitation, on account of the particular characteristics proper to any single earthquake record, but it is a clear trend that can be observed from the presented results for the mean responses to the five earthquake excitations used in this study. Although there is a few instances in Fig. 2 where the weak element ductility amplification ratios decreases for step increases in Ω, most of these decreases are of negligible magnitudes, and principally occur for low Ω values and large dissimilitudes in element yield strengths.

Based on previous observations, the following design recommendation can be formulated: For structural systems which can be idealized within the restrictions of this study, assuming the yield strength of the LLRSEs are dissimilar and can be estimated, the ductility demand of the weaker element is expected to exceed by approximately 50% the ductility demand of a SDOF of similar yield strength, if Ω is larger than 1.2. The designer expecting to limit the ductility demand on structural members in those cases should reduce its target ductility demand by 30% (1/1.5 = 0.67).

CONCLUSIONS

For the simple initially symmetric structures studied having unbalanced yield strengths in plan, a transient torsional response is created by the desynchronizing in inelastic element response, despite the existence of symmetry in the elastic domain. Resulting element ductility amplification ratios will remain low provided the ratio of uncoupled frequencies Ω is not excessively large (preferably 1.2 and lower) and the yield strength of the weaker element in the initially symmetric structure is not less than the yield strength of the equivalent SDOF. This conclusion is seen to remain valid for all translational periods and level of seismic excitation.

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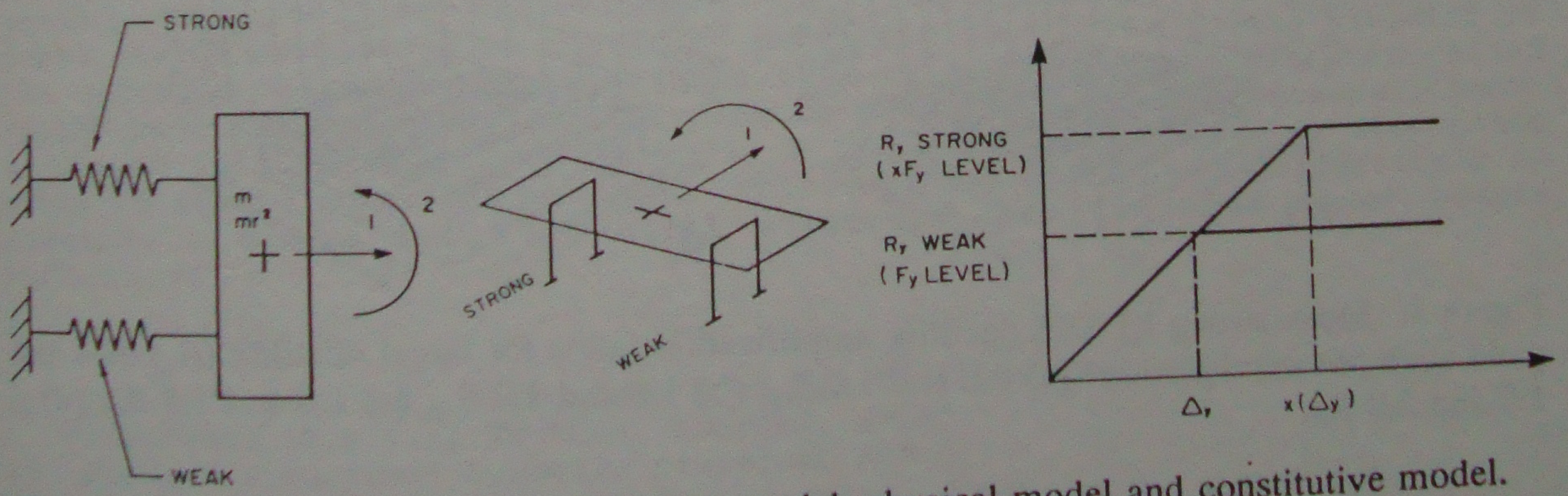

Figure 1: Two LLRSE system - Computer model, physical model and constitutive model.

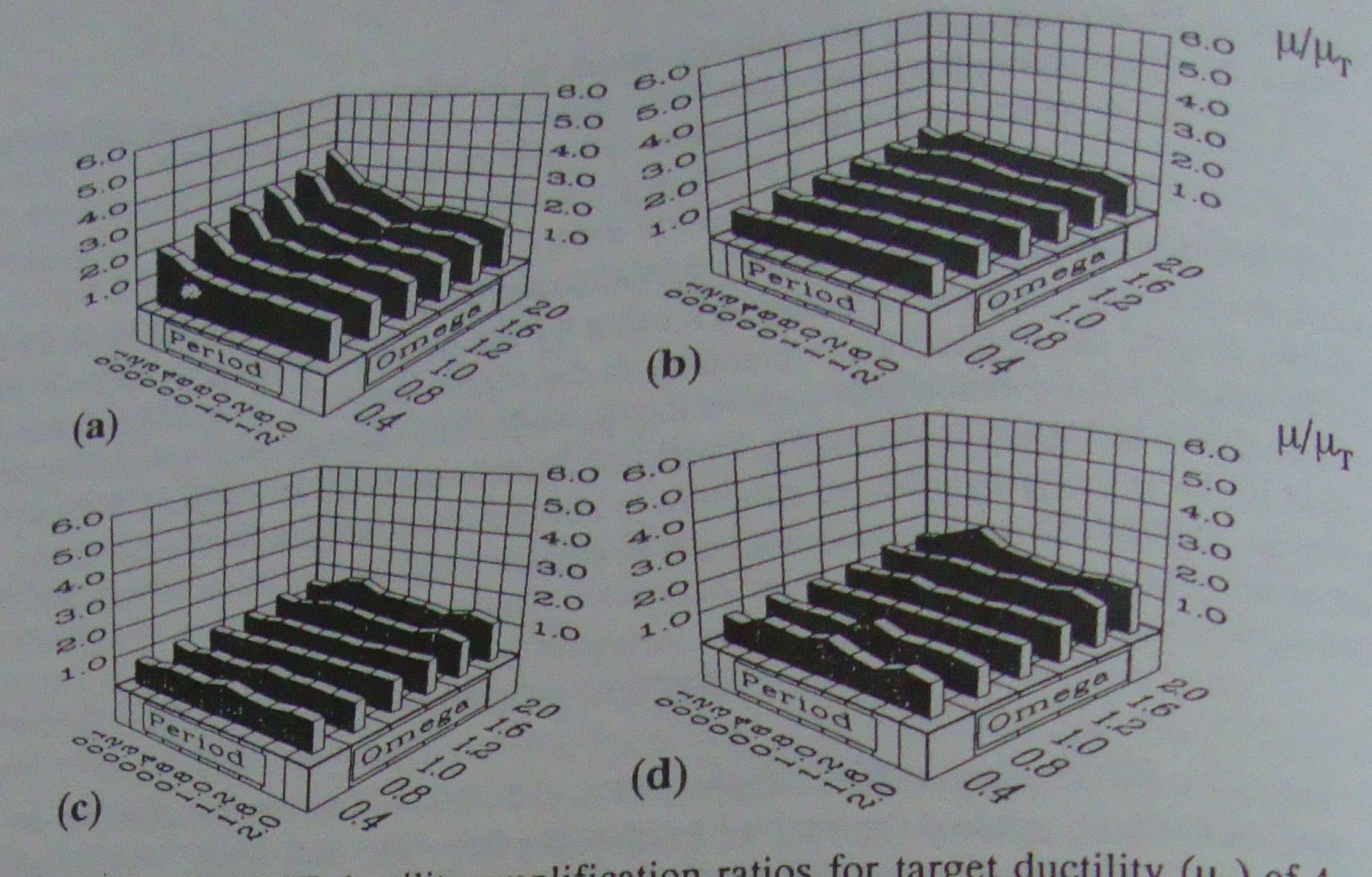

Figure 2: Mean weak LLRSE ductility amplification ratios for target ductility (μ_T) of 4 and yield strengths combinations (a) 0.8 and 1.0 R_Y, (b) 1.0 and 1.2 R_Y, (c) 1.0 and 1.5 R_Y, (d) 1.0 and 2.0 R_Y.

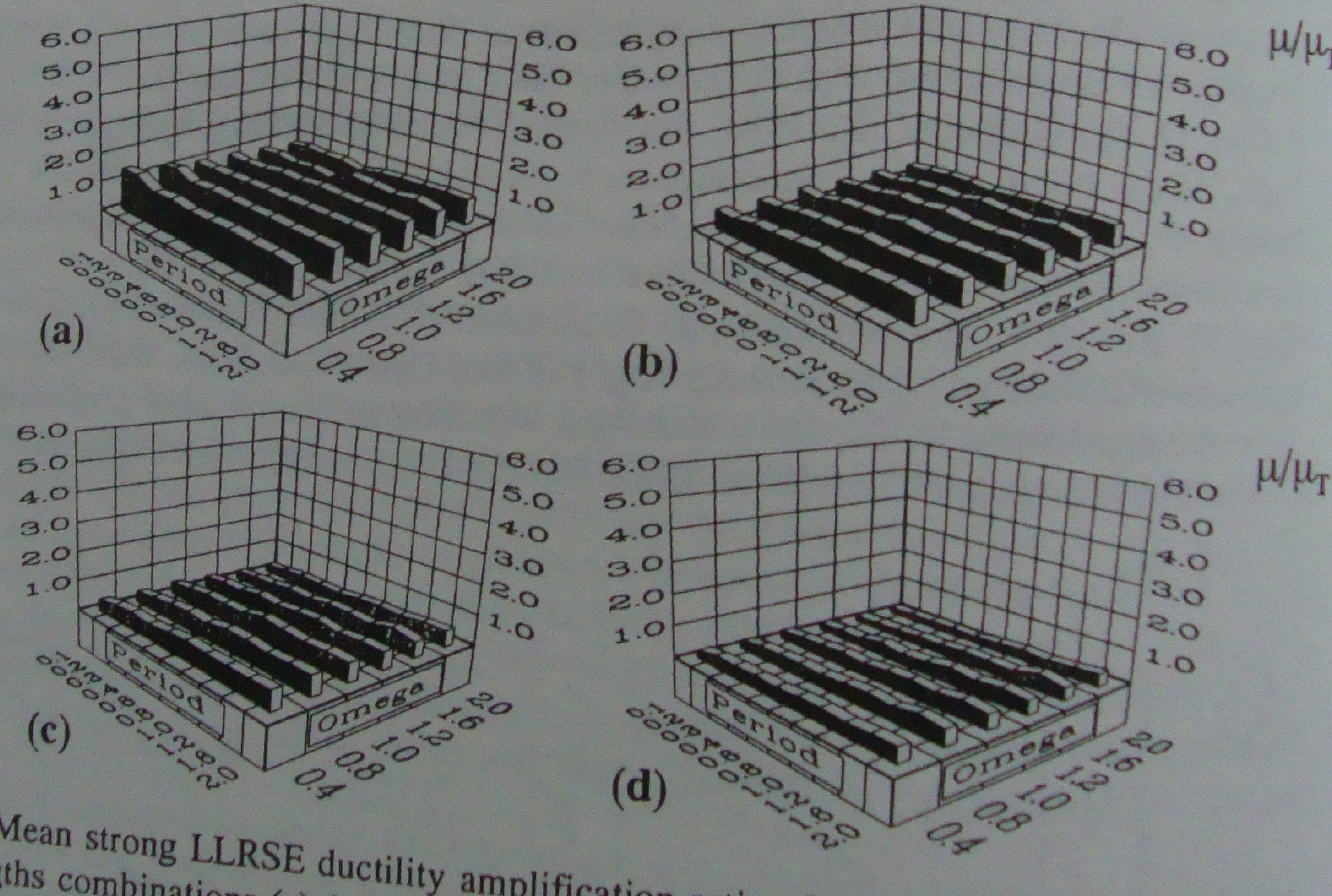

Figure 3: Mean strong LLRSE ductility amplification ratios for target ductility (μ_T) of 4 and yield strengths combinations (a) 0.8 and 1.0 R_Y, (b) 1.0 and 1.2 R_Y, (c) 1.0 and 1.5 R_Y, (d) 1.0 and 2.0 R_Y.